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DOI:

[10.1016/j.sigpro.2015.09.007](https://doi.org/10.1016/j.sigpro.2015.09.007)

Document Version

Peer reviewed version

[Link to publication record in King's Research Portal](#)

Citation for published version (APA):

Gao, Y., Li, H., Wu, L., Karimi, H. R., & Lam, H. K. (2016). Optimal control of discrete-time interval type-2 fuzzy-model-based systems with D-stability constraint and control saturation. *SIGNAL PROCESSING*, 120, 409-421. <https://doi.org/10.1016/j.sigpro.2015.09.007>

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Optimal Control of Discrete-Time Interval Type-2 Fuzzy-Model-Based Systems with \mathcal{D} -Stability Constraint and Control Saturation

Yabin Gao, Hongyi Li, Ligang Wu, Hamid Reza Karimi and Hak-Keung Lam

Abstract

This paper investigates optimal control problem for discrete-time interval type-2 (IT2) fuzzy systems with poles constraint. An IT2 fuzzy controller is characterized by two predefined functions, and the membership functions and the premise rules of the IT2 fuzzy controller can be chosen freely. The pole assignment is considered, which is constrained in a presented disk region. Based on Lyapunov stability theory, sufficient conditions of asymptotic stability with an \mathcal{H}_∞ performance are obtained for the discrete-time IT2 fuzzy model based (FMB) system. Based on the criterion, the desired IT2 state-feedback controller is designed to guarantee that the closed-loop system is asymptotically stable with a prescribed \mathcal{H}_∞ performance condition and all the poles rest in the disk region. Finally, a practical example is shown to illustrate the effectiveness of the presented design scheme.

Keywords: \mathcal{H}_∞ control; T-S fuzzy model; Interval type-2 fuzzy system; \mathcal{D} -stability; Input constraints.

This work was partially supported by the National Natural Science Foundation of China (61203002, 61304003, 61174126 and 61222301), the Program for New Century Excellent Talents in University (NCET-13-0696), the Program for Liaoning Innovative Research Team in University (LT2013023), the Program for Liaoning Excellent Talents in University (LR2013053), and the Fundamental Research Funds for the Central Universities (HIT.BRETIV.201303).

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I. INTRODUCTION

It is well known that the fuzzy set [1] can be represented by a membership (characteristic) function, which assigns to each object a grade of membership ranging between zero and one. Based on the fuzzy set, the fuzzy logic control problem has been extensively investigated in [2]–[8] for complex nonlinear systems. Furthermore, under the fuzzy sets, the Takagi-Sugeno (T-S) fuzzy model [9] can be described by a set of “IF–THEN” rules in the antecedents and linear time-invariant (LTI) dynamic systems in the consequent. It has been shown that the fuzzy set is able to capture nonlinearities in the system via an interpolation method [9], and some local linear systems with fuzzy weighting functions can be connected smoothly in a T-S fuzzy model. The well-known T-S fuzzy-model-based (FMB) approach [9], [10] was recognized as a very effective technique to represent nonlinear systems for further modeling, analyzing and controller designing. Recently, various stability and stabilization results have been reported for T-S FMB systems, see for example, [11]–[13], [13], [14], [14]–[25], and the references therein. To mention a few, the stability analysis and stabilization problems for discrete-time T-S fuzzy systems were investigated in [26]. The work in [27] solved the \mathcal{H}_∞ fuzzy control problem of a class of systems with repeated scalar nonlinearities represented by a modified T-S fuzzy model. The stabilization problem of T-S fuzzy systems of discrete-time form was addressed via static output-feedback controller in [28]. It should be pointed out that the above stability analysis and control synthesis results are all based on type-1 fuzzy set. In addition, note that the control problem for type-1 T-S fuzzy models become invalid if the membership functions contain uncertainties. As the parameter uncertainties [29], [30] often exist in nonlinear systems, the type-1 fuzzy model can not deal with the uncertainties completely. Thus, the parametric uncertainties hidden in membership functions can not be entirely utilized under the non-parallel distributed compensation design technique in the control design problem.

Once the nonlinear system subject to parametric uncertainties is considered, it will engender the uncertain grades of membership in value. More recently, based on type-2 fuzzy set, an interval type-2 (IT2) fuzzy model was presented and was used to handle the nonlinear plants subject to parameter uncertainties in [31]. It has been shown that the performance of IT2 fuzzy logic systems is superior than the type-1 fuzzy logic systems in the aspect of handling parametric uncertainties. The IT2 fuzzy models have attracted more attention and lots of control design results have been proposed, see for example, [32]–[40]. It is worth noting that the IT2 FMB control systems were developed firstly in [34], in which an IT2 T-S fuzzy model was proposed for representing the uncertain membership functions. It has been validated that such IT2 fuzzy state-feedback control method can achieve less conservative results than

the traditional type-1 non-parallel distributed compensation ones. Moreover, the work in [35] proposed a novel IT2 fuzzy controller, in which the number of fuzzy rules and the membership functions can be freely determined. However, it is found that there are no results on the IT2 fuzzy control design for discrete-time systems in existing literature. Therefore, inspired by [34] and [35], this paper will make a new attempt to model the discrete-time IT2 fuzzy systems and design IT2 fuzzy controller for such systems. On the other hand, a good controller should also deliver sufficiently well-damped and fast time responses in practice. However, due to parametric uncertainties originating from various sources, the exact pole location is not required. In order to guarantee a well-defined performance of the closed-loop system, it is necessary to locate the pole in a prescribed region. Therefore, in this paper, all the poles of the closed-loop system are assigned in a desired disk region such that satisfactory transients can be guaranteed for the discrete-time IT2 FMB systems.

Motivated by the above discussion, this paper focuses on designing a novel IT2 fuzzy \mathcal{H}_∞ state-feedback controller for discrete-time IT2 FMB systems with \mathcal{D} -stability constraint. Firstly, the discrete-time IT2 FMB systems and a fuzzy controller are constructed for control design. Secondly, using Lyapunov stability theory, new stability conditions are derived for the closed-loop systems, and then, the desired IT2 fuzzy state-feedback controller is designed such that the closed-loop system is asymptotically stable with an \mathcal{H}_∞ performance and all the poles are rested in the disk region. Finally, simulation results for the inverted pendulum model are employed to validate the effectiveness and usefulness of the presented design methods. The key contributions of this paper are as follows: 1) An IT2 fuzzy control system and an IT2 fuzzy controller of discrete-time are first constructed in this work. 2) It is not required that the discrete-time IT2 fuzzy control systems and fuzzy controller share the common lower membership functions and common upper membership functions, the membership functions and the premise rules of the IT2 fuzzy controller can be chosen freely. 3) The IT2 fuzzy state-feedback controller with an input constraint is designed for the discrete-time IT2 FMB control systems, which ensures that all the poles of the closed-loop system are rested in a given disk region.

The following construction of this paper is: Section II introduces the discrete-time IT2 FMB control systems and proposes the form of the IT2 fuzzy state-feedback controller. Section III presents the main results and Section IV provides simulation results to illustrate the effectiveness of the presented method. Section V concludes this paper.

Notation: $\ell_2[0, \infty)$ denotes the space of square-integrable vector functions over $[0, \infty)$. For $P \in \mathbb{R}^{n \times n}$, the $P > 0$ ($P \geq 0$) means that the matrix P is real symmetric positive definite (positive semi-definite). The symbol “*” in a matrix $P \in \mathbb{R}^{n \times n}$ stands for the transposed elements in the symmetric positions.

The superscripts “ T ” and “ -1 ” denote the matrix transpose and inverse, respectively. Identity matrix is denoted by “ I ” with appropriate dimensions. All matrices are assumed to be compatible dimensions for algebraic operations throughout the paper.

II. PROBLEM FORMULATION

Consider p -rule IT2 T-S fuzzy system of discrete-time form for representing the dynamics of a nonlinear plant below:

Fuzzy Rule i : IF $f_1(\xi(k))$ is M_1^i, \dots , and $f_\alpha(\xi(k))$ is M_α^i, \dots , and $f_\Theta(\xi(k))$ is M_Θ^i , THEN

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k) + B_{wi} w(k), \\ z(k) = C_i x(k) + D_i u(k) + D_{wi} w(k), \\ x(k) = \chi(k), \end{cases} \quad (1)$$

where M_α^i is an IT2 fuzzy set of i -th rule ($i = 1, 2, \dots, p$ and $\alpha = 1, 2, \dots, \Theta$); $f_\alpha(\xi(k))$ is the measurable premise variable; Θ is a positive integer; $x(k) \in \mathbb{R}^n$ stands for the system state vector; $u(k) \in \mathbb{R}^v$ is the control input; $w(k) \in \mathbb{R}^s$ is a disturbance input belonging to $\ell_2[0, \infty)$; $z(k) \in \mathbb{R}^m$ stands for the controlled output, and $\chi(k)$ is a continuous vector-valued initial function. $A_i, B_i, B_{wi}, C_i, D_i$ and D_{wi} are system matrices with appropriate dimensions. The i -th interval sets of firing strength is:

$$\Phi_i(\xi(k)) = [\underline{\phi}_i(\xi(k)), \bar{\phi}_i(\xi(k))]$$

where

$$\begin{aligned} \underline{\phi}_i(\xi(k)) &= \prod_{\alpha=1}^{\Theta} \underline{\mu}_{M_\alpha^i}(f_\alpha(\xi(k))) \geq 0, \\ \bar{\phi}_i(\xi(k)) &= \prod_{\alpha=1}^{\Theta} \bar{\mu}_{M_\alpha^i}(f_\alpha(\xi(k))) \geq 0, \end{aligned}$$

$\bar{\mu}_{M_\alpha^i}(f_\alpha(\xi(k))) \geq \underline{\mu}_{M_\alpha^i}(f_\alpha(\xi(k))) \geq 0$, and $\bar{\phi}_i(\xi(k)) \geq \underline{\phi}_i(\xi(k)) \geq 0$, in which $\underline{\mu}_{M_\alpha^i}(f_\alpha(\xi(k))) \in [0, 1]$ and $\bar{\mu}_{M_\alpha^i}(f_\alpha(\xi(k))) \in [0, 1]$ are the lower and upper membership functions, respectively. $\underline{\phi}_i(\xi(k))$ and $\bar{\phi}_i(\xi(k))$ stands the lower and upper grades of membership, respectively. Then the discrete-time IT2

T-S fuzzy system in (1) can be defined as:

$$\begin{cases} x(k+1) = \sum_{i=1}^p \phi_i(\xi(k)) (A_i x(k) + B_i u(k) + B_{wi} w(k)), \\ z(k) = \sum_{i=1}^p \phi_i(\xi(k)) (C_i x(k) + D_i u(k) + D_{wi} w(k)), \\ x(k) = \chi(k), \end{cases} \quad (2)$$

where

$$\begin{aligned}\phi_i(\xi(k)) &= \underline{\alpha}_i(\xi(k)) \underline{\phi}_i(\xi(k)) + \overline{\alpha}_i(\xi(k)) \overline{\phi}_i(\xi(k)) \geq 0, \\ 0 &\leq \underline{\alpha}_i(\xi(k)) \leq 1, \quad 0 \leq \overline{\alpha}_i(\xi(k)) \leq 1, \\ 1 &= \underline{\alpha}_i(\xi(k)) + \overline{\alpha}_i(\xi(k)),\end{aligned}\tag{3}$$

in which $\underline{\alpha}_i(\xi(k))$ and $\overline{\alpha}_i(\xi(k))$ are weighting coefficient functions that not necessarily be determined in real systems. $\phi_i(\xi(k))$ denote the grades of membership of the real membership functions. Similar to the form of fuzzy state-feedback controller proposed in [35], the fuzzy state-feedback controller with c rules is represented as:

Plant Rule j : IF $g_1(\xi(k))$ is N_1^j, \dots , and $g_\beta(\xi(k))$ is N_β^j, \dots , and $g_\Omega(\xi(k))$ is N_Ω^j , THEN

$$u(k) = G_j x(k),\tag{4}$$

where N_β^j denotes an IT2 fuzzy set of j -th rule ($j = 1, 2, \dots, c$ and $\beta = 1, 2, \dots, \Omega$); Ω is a positive integer; $g_\beta(\xi(k))$ is the measurable premise variable, and G_j stands for the control gain to be designed.

The interval sets of firing strength of the j -th rule is:

$$\Psi_j(\xi(k)) = [\underline{\psi}_j(\xi(k)), \overline{\psi}_j(\xi(k))]$$

where

$$\begin{aligned}\underline{\psi}_j(\xi(k)) &= \prod_{\beta=1}^{\Omega} \underline{\mu}_{N_\beta^j}(g_\beta(\xi(k))) \geq 0, \\ \overline{\psi}_j(\xi(k)) &= \prod_{\beta=1}^{\Omega} \overline{\mu}_{N_\beta^j}(g_\beta(\xi(k))) \geq 0,\end{aligned}$$

$\overline{\mu}_{N_\beta^j}(g_\beta(\xi(k))) \geq \underline{\mu}_{N_\beta^j}(g_\beta(\xi(k))) \geq 0$, and $\overline{\psi}_j(\xi(k)) \geq \underline{\psi}_j(\xi(k)) \geq 0$, in which $\underline{\mu}_{N_\beta^j}(g_\beta(\xi(k))) \in [0, 1]$ and $\overline{\mu}_{N_\beta^j}(g_\beta(\xi(k))) \in [0, 1]$ are the lower and upper membership functions, and $\underline{\psi}_j(\xi(k))$ and $\overline{\psi}_j(\xi(k))$ are the lower and upper grades of membership, respectively. Then the fuzzy controller in (4) is expressed as:

$$u(k) = \sum_{j=1}^c \psi_j(\xi(k)) G_j x(k),\tag{5}$$

where for $j = 1, 2, \dots, c$,

$$\begin{aligned}\psi_j(\xi(k)) &= \frac{\underline{\beta}_j(\xi(k)) \underline{\psi}_j(\xi(k)) + \overline{\beta}_j(\xi(k)) \overline{\psi}_j(\xi(k))}{\sum_{\kappa=1}^c (\underline{\beta}_\kappa(\xi(k)) \underline{\psi}_\kappa(\xi(k)) + \overline{\beta}_\kappa(\xi(k)) \overline{\psi}_\kappa(\xi(k)))} \geq 0, \\ 0 &\leq \underline{\beta}_j(\xi(k)) \leq 1, \quad 0 \leq \overline{\beta}_j(\xi(k)) \leq 1, \\ 1 &= \underline{\beta}_j(\xi(k)) + \overline{\beta}_j(\xi(k)),\end{aligned}\tag{6}$$

in which $\underline{\beta}_j(\xi(k))$ and $\bar{\beta}_j(\xi(k))$ are predefined functions; $\psi_j(\xi(k))$ denote the grades of membership of the real membership functions. From (2) and (5), we have

$$\sum_{i=1}^p \phi_i(\xi(k)) = \sum_{j=1}^c \psi_j(\xi(k)) = \sum_{i=1}^p \sum_{j=1}^c \phi_i(\xi(k)) \psi_j(\xi(k)) = 1.$$

Thus, the closed-loop IT2 fuzzy control system is rewritten as:

$$\begin{cases} x(k+1) = \sum_{i=1}^p \sum_{j=1}^c \phi_i(\xi(k)) \psi_j(\xi(k)) [(A_i + B_i G_j) x(k) + B_{wi} w(k)], \\ z(k) = \sum_{i=1}^p \sum_{j=1}^c \phi_i(\xi(k)) \psi_j(\xi(k)) [(C_i + D_i G_j) x(k) + D_{wi} w(k)]. \end{cases} \quad (7)$$

Thus, any membership functions within the footprint of uncertainty (FOU) [34] can be reconstructed by the lower and upper membership functions. In order to facilitate the main results in the next section, according to [35], we divide the state space and the FOU for developing the main results. The description of them is detailed as follows:

- The state space Δ is divided into q sub-state spaces Δ_k ($k = 1, 2, \dots, q$), such that $\Delta = \bigcup_{k=1}^q \Delta_k$.
- Furthermore, to consider more information in membership functions, local lower and upper membership functions within the FOU are introduced. The FOU is divided into $\vartheta + 1$ sub-FOUs. For $l = 1, 2, \dots, \vartheta + 1$, the lower and upper membership functions in the l -th sub-FOU are defined, respectively, as follows for $\forall i, j, k, l$:

$$\underline{h}_{ijl}(\xi(k)) = \sum_{k=1}^q \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r,kl}(x_r(k)) \underline{\epsilon}_{ij i_1 i_2 \dots i_n kl}, \quad (8)$$

$$\bar{h}_{ijl}(\xi(k)) = \sum_{k=1}^q \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r,kl}(x_r(k)) \bar{\epsilon}_{ij i_1 i_2 \dots i_n kl}, \quad (9)$$

where $\underline{\epsilon}_{ij i_1 i_2 \dots i_n kl}$ and $\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl}$ are constants to be determined, and $0 \leq \underline{\epsilon}_{ij i_1 i_2 \dots i_n kl} \leq \bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} \leq 1$; for $r, s = 1, 2, \dots, n$, $i_r, i_s = 1, 2$, $k = 1, 2, \dots, q$, and $x(k) \in \Delta_k$, we have $0 \leq v_{ri_r,kl}(x_r(k)) \leq 1$ and $v_{r1kl}(x_r(k)) + v_{r2kl}(x_r(k)) = 1$; and $v_{ri_r,kl}(x_r(k)) = 0$ if otherwise. Thus, for all l ,

$$\sum_{k=1}^q \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r,kl}(x_r(k)) = 1. \quad (10)$$

Then, we rewrite the IT2 FMB control system (7) as follows:

$$\begin{cases} x(k+1) = \sum_{i=1}^p \sum_{j=1}^c h_{ij}(\xi(k)) [(A_i + B_i G_j) x(k) + B_{wi} w(k)], \\ z(k) = \sum_{i=1}^p \sum_{j=1}^c h_{ij}(\xi(k)) [(C_i + D_i G_j) x(k) + D_{wi} w(k)], \end{cases} \quad (11)$$

where

$$\begin{aligned} h_{ij}(\xi(k)) &\triangleq \phi_i(\xi(k)) \psi_j(\xi(k)) \\ &= \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) (\underline{\delta}_{ijl}(\xi(k)) \underline{h}_{ijl}(\xi(k)) + \bar{\delta}_{ijl}(\xi(k)) \bar{h}_{ijl}(\xi(k))) \end{aligned}$$

with $\sum_{i=1}^p \sum_{j=1}^c h_{ij}(\xi(k)) = 1$, the two functions $0 \leq \underline{\delta}_{ijl}(\xi(k)) \leq \bar{\delta}_{ijl}(\xi(k)) \leq 1$ satisfy the property $\underline{\delta}_{ijl}(\xi(k)) + \bar{\delta}_{ijl}(\xi(k)) = 1$, and are not necessarily known, and $\sigma_{ijl}(\xi(k)) = 1$ if the membership function $h_{ijl}(\xi(k))$ is within the l -th sub-FOU; otherwise, $\sigma_{ijl}(\xi(k)) = 0$.

Remark 1: For the discrete-time IT2 fuzzy systems in (2), our purpose is to confirm the system stability with an \mathcal{H}_∞ performance by determining the feedback gains G_j . Based on closed-loop system (11), sufficient conditions of the stability for the discrete-time IT2 fuzzy control system (7) with \mathcal{H}_∞ performance can be derived in the next section.

The following lemma is stated for the main results.

Lemma 1: [41] The eigenvalues of a given matrix $\mathcal{A} \in \mathbb{R}^{n \times n}$ belong to the closed circular region $\mathcal{D}(\varrho, \tau) \in \mathcal{D}(0, 1)$ with center ϱ and radius τ , if and only if there exists a symmetric positive-definite matrix $\mathcal{P} \in \mathbb{R}^{n \times n}$ such that

$$\begin{bmatrix} -\mathcal{P} & \mathcal{P}(\mathcal{A} - \varrho I_n) \\ * & -\tau^2 \mathcal{P} \end{bmatrix} \leq 0.$$

The control objective of this paper is to design an IT2 fuzzy state-feedback controller such that system (7) is asymptotically stable and satisfies a prescribed \mathcal{H}_∞ performance and \mathcal{D} -stability, simultaneously, under input constraint. The detailed requirements are listed as follows:

- 1) The closed-loop system in (11) is asymptotically stable.
- 2) The disturbance input $w(k)$ to system output $z(k)$ is attenuated below a desired level in the \mathcal{H}_∞ sense that is, for a given scalar $\gamma > 0$, under zero initial condition, it holds that

$$\|z\|_2 < \gamma \|w\|_2, \quad \forall 0 \neq w \in \ell_2[0, \infty), \quad (12)$$

where $\|z\|_2 = \sqrt{\sum_{k=0}^{\infty} z^T(k) z(k)}$.

- 3) The control input is subject to constraint of $|u(k)| \leq u_{\max}$ (u_{\max} is a given positive scalar).
- 4) \mathcal{D} -stability, specifically, the eigenvalues of system matrices $\mathcal{A}_{ij} = \overbrace{A_i + B_i G_j} \in \mathbb{R}^{n \times n}$ all belong to the closed disk region $\mathcal{D}(\varrho, \tau)$ (ϱ is a given negative scalar denoting the center of the disk; τ is a given positive scalar denoting the radius of the disk).

III. MAIN RESULTS

In this section, by using Lyapunov functional approach, a sufficient criterion is first given to satisfy the above-mentioned four requirements.

Theorem 1: Considering the discrete-time IT2 FMB system in (11) with FOU being divided into $\vartheta + 1$ sub-FOUs and the state space being divided into q connected sub-state spaces, for given input constraint $u_{\max} > 0$, scalar $\rho > 0$, and disk region $\mathcal{D}(\varrho, \tau)$ and the scalar $\gamma > 0$, system (11) with the input constraint is asymptotically stable with an \mathcal{H}_∞ performance index γ with all the poles resting in the disk region $\mathcal{D}(\varrho, \tau)$, if there exist symmetric matrices $P > 0$, $X_{ijl} > 0$, $Y_{ijl} > 0$, $U_{ijl} > 0$, $V_{ijl} > 0$, $R_{ijl} > 0$ ($i = 1, 2, \dots, p$, $j = 1, 2, \dots, c$, $l = 1, 2, \dots, \vartheta + 1$) and matrix Q such that:

$$\forall i_1, i_2, \dots, i_n, k, l, \sum_{i=1}^p \sum_{j=1}^c Z_{ij} - Q < 0, \quad (13)$$

$$\forall i, j, l, \Xi_{2ijl} + R_{ijl} + Q > 0, \quad (14)$$

$$\forall i, j, l, P - X_{ijl} < 0, \quad (15)$$

$$\forall i, j, l, I - Y_{ijl} < 0, \quad (16)$$

$$\forall j, \begin{bmatrix} -\rho^{-1}I & G_j \\ * & -u_{\max}^2 P \end{bmatrix} < 0, \quad (17)$$

$$\forall i, j, \begin{bmatrix} -P & P(A_{ij} - \varrho I) \\ * & -\tau^2 P \end{bmatrix} < 0, \quad (18)$$

where

$$\begin{aligned} \Xi_{1ijl} &\triangleq \begin{bmatrix} A_{ij}^T \Upsilon_{11} A_{ij} + C_{ij}^T \Upsilon_{12} C_{ij} - P & A_{ij}^T \Upsilon_{11} B_{wi} + C_{ij}^T \Upsilon_{12} D_{wi} \\ * & B_{wi}^T \Upsilon_{11} B_{wi} + D_{wi}^T \Upsilon_{12} D_{wi} - \gamma^2 I \end{bmatrix}, \\ \Xi_{2ijl} &\triangleq \begin{bmatrix} A_{ij}^T \Upsilon_{21} A_{ij} + C_{ij}^T \Upsilon_{22} C_{ij} - P & A_{ij}^T \Upsilon_{21} B_{wi} + C_{ij}^T \Upsilon_{22} D_{wi} \\ * & B_{wi}^T \Upsilon_{21} B_{wi} + D_{wi}^T \Upsilon_{22} D_{wi} - \gamma^2 I \end{bmatrix}, \\ Z_{ij} &\triangleq \bar{\epsilon}_{ij i_1 i_2 \dots i_n k l} \Xi_{1ijl} - (\epsilon_{ij i_1 i_2 \dots i_n k l} - \bar{\epsilon}_{ij i_1 i_2 \dots i_n k l}) R_{ijl} + \bar{\epsilon}_{ij i_1 i_2 \dots i_n k l} Q, \\ A_{ij} &\triangleq A_i + B_i G_j, \quad C_{ij} \triangleq C_i + D_i G_j, \\ \Upsilon_{11} &\triangleq P + U_{ijl}, \quad \Upsilon_{12} \triangleq I + V_{ijl}, \quad \Upsilon_{21} \triangleq P - X_{ijl}, \quad \Upsilon_{22} \triangleq I - Y_{ijl}. \end{aligned}$$

Proof: Considering some slack matrices in the following inequalities under the *S-procedure* [42]:

$$\left[\sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) (\underline{\delta}_{ijl}(\xi(k)) \underline{h}_{ijl}(\xi(k)) + \bar{\delta}_{ijl}(\xi(k)) \bar{h}_{ijl}(\xi(k))) - 1 \right] Q = 0, \quad (19)$$

$$- \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) (1 - \underline{\delta}_{ijl}(\xi(k))) (\underline{h}_{ijl}(\xi(k)) - \bar{h}_{ijl}(\xi(k))) R_{ijl} \geq 0, \quad (20)$$

$$\sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) \bar{h}_{ijl}(\xi(k)) U_{ijl} \geq 0, \quad (21)$$

$$- \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) \underline{\delta}_{ijl}(\xi(k)) (\underline{h}_{ijl}(\xi(k)) - \bar{h}_{ijl}(\xi(k))) X_{ijl} \geq 0, \quad (22)$$

$$\sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) \bar{h}_{ijl}(\xi(k)) V_{ijl} \geq 0, \quad (23)$$

$$- \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) \underline{\delta}_{ijl}(\xi(k)) (\underline{h}_{ijl}(\xi(k)) - \bar{h}_{ijl}(\xi(k))) Y_{ijl} \geq 0. \quad (24)$$

Choosing Lyapunov function $V(k)$ for the system in (11) as follows:

$$V(k) = x^T(k) P x(k), \quad (25)$$

where symmetric matrices $P > 0$. From the system (11) and the function (25), based on inequalities (19)–(22), we have

$$\begin{aligned} \Delta V(k) &= x^T(k+1) P x(k+1) - x^T(k) P x(k) \\ &= \sum_{i=1}^p \sum_{j=1}^c \sum_{\kappa=1}^p \sum_{\iota=1}^c h_{ij}(\xi(k)) h_{\kappa\iota}(\xi(k)) [A_{ij}x(k) + B_{wi}w(k)]^T \\ &\quad \times P [A_{\kappa\iota}x(k) + B_{w\kappa}w(k)] - x^T(k) P x(k) \\ &\leq \frac{1}{2} \sum_{i=1}^p \sum_{j=1}^c \sum_{\kappa=1}^p \sum_{\iota=1}^c h_{ij}(\xi(k)) h_{\kappa\iota}(\xi(k)) \left\{ [A_{ij}x(k) + B_{wi}w(k)]^T P [A_{ij}x(k) + B_{wi}w(k)] \right. \\ &\quad \left. + [A_{\kappa\iota}x(k) + B_{w\kappa}w(k)]^T P [A_{\kappa\iota}x(k) + B_{w\kappa}w(k)] \right\} - x^T(k) P x(k) \end{aligned}$$

$$\begin{aligned}
&= \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) (\underline{\delta}_{ijl}(\xi(k)) \underline{h}_{ijl}(\xi(k)) + \bar{\delta}_{ijl}(\xi(k)) \bar{h}_{ijl}(\xi(k))) \eta^T(k) \tilde{\Xi}_{0ijl} \eta(k) \\
&\leq \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) (\underline{\delta}_{ijl}(\xi(k)) \underline{h}_{ijl}(\xi(k)) + \bar{\delta}_{ijl}(\xi(k)) \bar{h}_{ijl}(\xi(k))) \eta^T(k) \tilde{\Xi}_{0ijl} \eta(k) \\
&\quad + \left[\sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) (\underline{\delta}_{ijl}(\xi(k)) \underline{h}_{ijl}(\xi(k)) + \bar{\delta}_{ijl}(\xi(k)) \bar{h}_{ijl}(\xi(k))) - 1 \right] \eta^T(k) Q \eta(k) \\
&\quad - \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) (1 - \underline{\delta}_{ijl}(\xi(k))) (\underline{h}_{ijl}(\xi(k)) - \bar{h}_{ijl}(\xi(k))) \eta^T(k) R_{ijl} \eta(k) \\
&\quad + \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) \bar{h}_{ijl}(\xi(k)) x^T(k+1) U_{ijl} x(k+1) \\
&\quad - \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) \underline{\delta}_{ijl}(\xi(k)) (\underline{h}_{ijl}(\xi(k)) - \bar{h}_{ijl}(\xi(k))) x^T(k+1) X_{ijl} x(k+1) \\
&= \eta^T(k) \left\{ \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) \left[\bar{h}_{ijl} \tilde{\Xi}_{1ijl} - (\underline{h}_{ijl} - \bar{h}_{ijl}) R_{ijl} + \bar{h}_{ijl} Q \right] - Q \right\} \eta(k) \\
&\quad + \eta^T(k) \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) \underline{\delta}_{ijl}(\xi(k)) (\underline{h}_{ijl} - \bar{h}_{ijl}) \left(\tilde{\Xi}_{2ijl} + R_{ijl} + Q \right) \eta(k), \tag{26}
\end{aligned}$$

where $\eta^T(k) \triangleq \begin{bmatrix} x^T(k) & w^T(k) \end{bmatrix}$, and

$$\begin{aligned}
\tilde{\Xi}_{0ijl} &\triangleq \begin{bmatrix} A_{ij}^T P A_{ij} - P & A_{ij}^T P B_{wi} \\ * & B_{wi}^T P B_{wi} \end{bmatrix}, \\
\tilde{\Xi}_{1ijl} &\triangleq \begin{bmatrix} A_{ij}^T (P + U_{ijl}) A_{ij} - P & A_{ij}^T (P + U_{ijl}) B_{wi} \\ * & B_{wi}^T (P + U_{ijl}) B_{wi} \end{bmatrix}, \\
\tilde{\Xi}_{2ijl} &\triangleq \begin{bmatrix} A_{ij}^T (P - X_{ijl}) A_{ij} - P & A_{ij}^T (P - X_{ijl}) B_{wi} \\ * & B_{wi}^T (P - X_{ijl}) B_{wi} \end{bmatrix}.
\end{aligned}$$

When considering $w(k) = 0$, we know that $\Delta V(k) < 0$ from (13)–(14). Then, considering \mathcal{H}_∞ performance in (12) with (22), under the zero initial condition, we have

$$\begin{aligned}
J &= \sum_{k=0}^{\infty} [z^T(k) z(k) - \gamma^2 w^T(k) w(k)] \leq J + V(\infty) - V(0) \\
&= \sum_{k=0}^{\infty} [z^T(k) z(k) - \gamma^2 w^T(k) w(k) + \Delta V(k)] \tag{27}
\end{aligned}$$

$$\begin{aligned}
&\leq \sum_{k=0}^{\infty} \left\{ \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) (\underline{\delta}_{ijl}(\xi(k)) \underline{h}_{ijl}(\xi(k)) + \bar{\delta}_{ijl}(\xi(k)) \bar{h}_{ijl}(\xi(k))) \right. \\
&\quad \times \eta^T(k) \begin{bmatrix} C_{ij}^T C_{ij} & C_{ij}^T D_{wi} \\ * & D_{wi}^T D_{wi} - \gamma^2 I \end{bmatrix} \eta(k) + \Delta V(k) \\
&\quad + \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) \bar{h}_{ijl}(\xi(k)) z^T(k) V_{ijl} z(k) \\
&\quad \left. - \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) \underline{\delta}_{ijl}(\xi(k)) (\underline{h}_{ijl}(\xi(k)) - \bar{h}_{ijl}(\xi(k))) z^T(k) Y_{ijl} z(k) \right\} \\
&= \sum_{k=0}^{\infty} \eta^T(k) \left\{ \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) [\bar{h}_{ijl} \Xi_{1ijl} - (\underline{h}_{ijl} - \bar{h}_{ijl}) R_{ijl} + \bar{h}_{ijl} Q] - Q \right\} \eta(k) \\
&\quad + \sum_{k=0}^{\infty} \eta^T(k) \sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) \underline{\delta}_{ijl}(\xi(k)) (\underline{h}_{ijl} - \bar{h}_{ijl}) (\Xi_{2ijl} + R_{ijl} + Q) \eta(k). \quad (28)
\end{aligned}$$

Thus, $J < 0$ in (28) can be obtained from the two sets of inequalities: $\Xi_{2ijl} + R_{ijl} + Q > 0$ (which is guaranteed by (14)), and

$$\sum_{i=1}^p \sum_{j=1}^c \sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) [\bar{h}_{ijl} \Xi_{1ijl} - (\underline{h}_{ijl} - \bar{h}_{ijl}) R_{ijl} + \bar{h}_{ijl} Q] - Q < 0. \quad (29)$$

Notice that only one $\sigma_{ijl}(\xi(k)) = 1$ for each fixed combination of i and j at any time, and $\sum_{l=1}^{\vartheta+1} \sigma_{ijl}(\xi(k)) = 1$, the inequalities in (29) is held by

$$\sum_{i=1}^p \sum_{j=1}^c [\bar{h}_{ijl} \Xi_{1ijl} - (\underline{h}_{ijl} - \bar{h}_{ijl}) R_{ijl} + \bar{h}_{ijl} Q] - Q < 0. \quad (30)$$

Considering \underline{h}_{ijl} in (8), \bar{h}_{ijl} in (9), and the equalities in (10), the following inequalities are equivalent to the inequalities in (30),

$$\sum_{k=1}^q \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r,kl}(x_r(k)) \left(\sum_{i=1}^p \sum_{j=1}^c Z_{ij} - Q \right) < 0. \quad (31)$$

Obviously, the inequalities in (31) is fulfilled by (13). Hence, $\|z\|_2 < \gamma \|w\|_2$ as $J < 0$. Thus, for all nonzero $w = w(k) \in \ell_2[0, \infty)$, the conditions (13)–(18) can guarantee the asymptotic stability of system (11) with an \mathcal{H}_∞ performance index γ .

In the following, the condition of the input constraint $|u(k)| \leq u_{\max}$ is derived. From the inequality in (27), we know

$$\Delta V(k) - \gamma^2 w^T(k) w(k) + z^T(k) z(k) < 0.$$

Thus, $\Delta V(k) - \gamma^2 w^T(k) w(k) < 0$, which implies

$$V(k) - V(0) < \gamma^2 \sum_{i=0}^k w^T(i) w(i) < \gamma^2 \|w\|_2^2. \quad (32)$$

Considering $V(k) = x^T(k) P x(k) > 0$, it can be concluded that

$$x^T(k) P x(k) = V(k) < \gamma^2 \|w\|_2^2 + V(0) < \rho = \gamma^2 w_{\max} + V(0),$$

where $w_{\max} = \frac{\rho - V(0)}{\gamma^2}$ denotes the disturbance energy bound. Then, it follows that

$$\begin{aligned} \max_{k>0} |u(k)|^2 &= \max_{k>0} \left\| \left[\sum_{j=1}^c \psi_j(\xi(k)) G_j x(k) \right]^T \left[\sum_{j=1}^c \psi_j(\xi(k)) G_j x(k) \right] \right\|_2 \\ &= \max_{k>0} \left\| x^T(k) P^{\frac{1}{2}} P^{-\frac{1}{2}} G_j^T G_j P^{-\frac{1}{2}} P^{\frac{1}{2}} x(k) \right\|_2 \\ &< \rho \cdot \lambda_{\max} \left(P^{-\frac{1}{2}} G_j^T G_j P^{-\frac{1}{2}} \right), \quad j = 1, 2, \dots, c, \end{aligned}$$

where $\lambda_{\max}(\cdot)$ represents maximal eigenvalue. From the above inequalities, we know that the input constraint is satisfied, if

$$\rho \cdot P^{-\frac{1}{2}} G_j^T G_j P^{-\frac{1}{2}} < u_{\max}^2 I. \quad (33)$$

By Schur complement, (33) is equivalent to (17).

In addition, considering the pole assignment for the closed-loop system in (11), by applying Lemma 2, we know the condition in (18) is satisfied such that the system in (11) is \mathcal{D} -stable. The proof is completed. \blacksquare

Remark 2: Theorem 1 provides a sufficient criterion of asymptotic stability for discrete-time IT2 fuzzy system in (11) with input constrain, \mathcal{H}_{∞} performance and \mathcal{D} -stability. If the disturbance input, the input constraint and \mathcal{D} -stability constraint are not considered in this paper, the stability condition can be also presented directly from Theorem 1.

The following theorem will present the existence condition of the IT2 fuzzy controller in the form of (4).

Theorem 2: Considering the discrete-time IT2 FMB system in (11) with FOU being divided into $\vartheta + 1$ sub-FOUs and the state space being divided into q connected substate spaces, for given input constraint $u_{\max} > 0$, scalar $\rho > 0$, and disk region $\mathcal{D}(\varrho, \tau)$, and the scalar $\gamma > 0$, system (11) with the input constraint is asymptotically stable with an \mathcal{H}_{∞} performance index γ with all the poles resting in the disk region $\mathcal{D}(\varrho, \tau)$, if there exist symmetric matrices $\bar{P} > 0$, $\bar{R}_{1ijl} > 0$, $\bar{R}_{2ijl} > 0$, $\bar{R}_{3ijl} > 0$ ($i = 1, 2, \dots, p$,

$j = 1, 2, \dots, c, l = 1, 2, \dots, \vartheta + 1$, $\bar{X}_{ijl}, \bar{Y}_{ijl}, \bar{U}_{ijl}, \bar{V}_{ijl}, \bar{Q}_1, \bar{Q}_2$ and \bar{Q}_3 satisfy

$$\forall i, j, i_1, i_2, \dots, i_n, k, l, \begin{bmatrix} \bar{\Gamma}_{1ijl} & \bar{\Gamma}_{2ijl} & \varpi \left(\bar{P} A_i^T + \bar{K}_j^T B_i^T \right) & \varpi \left(\bar{P} C_i^T + \bar{K}_j^T D_i^T \right) \\ * & \bar{\Gamma}_{3ijl} & \varpi B_{wi}^T & \varpi D_{wi}^T \\ * & * & -\bar{U}_{ijl} & 0 \\ * & * & * & -\bar{V}_{ijl} \end{bmatrix} < 0, (34)$$

$$\forall i, j, l, \begin{bmatrix} -\bar{P} + \bar{R}_{1ijl} + \bar{Q}_1 & \bar{R}_{2ijl} + \bar{Q}_2 & \bar{P} A_i^T + \bar{K}_j^T B_i^T & \bar{P} C_i^T + \bar{K}_j^T D_i^T \\ * & -\gamma^2 I + \bar{R}_{3ijl} + \bar{Q}_3 & B_{wi}^T & D_{wi}^T \\ * & * & \bar{X}_{ijl} & 0 \\ * & * & * & \bar{Y}_{ijl} \end{bmatrix} > 0, (35)$$

$$\forall j, \begin{bmatrix} -\rho^{-1} I & \bar{K}_j \\ * & -u_{\max}^2 \bar{P} \end{bmatrix} < 0, (36)$$

$$\forall i, j, \begin{bmatrix} -\bar{P} & A_i \bar{P} + B_i \bar{K}_j - \varrho \bar{P} \\ * & -\tau^2 \bar{P} \end{bmatrix} < 0, (37)$$

where

$$\begin{aligned} \bar{\Gamma}_{1ijl} &\triangleq -\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} \bar{P} + (\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \underline{\epsilon}_{ij i_1 i_2 \dots i_n kl}) \bar{R}_{1ijl} + \left(\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \frac{1}{pc} \right) \bar{Q}_1, \\ \bar{\Gamma}_{2ijl} &\triangleq (\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \underline{\epsilon}_{ij i_1 i_2 \dots i_n kl}) \bar{R}_{2ijl} + \left(\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \frac{1}{pc} \right) \bar{Q}_2, \quad \varpi \triangleq \sqrt{\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl}}, \\ \bar{\Gamma}_{3ijl} &\triangleq -\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} \gamma^2 I + (\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \underline{\epsilon}_{ij i_1 i_2 \dots i_n kl}) \bar{R}_{3ijl} + \left(\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \frac{1}{pc} \right) \bar{Q}_3. \end{aligned}$$

Moreover, if the above LMIs are feasible, then the control gains in (4) are

$$G_j = \bar{K}_j \bar{P}^{-1}. \quad (38)$$

Proof: Letting the following matrices

$$\bar{P} = P^{-1}, \quad \bar{K}_j = G_j P^{-1}, \quad \bar{R}_{1ijl} = P^{-T} R_{1ijl} P^{-1}, \quad \bar{R}_{2ijl} = P^{-T} R_{2ijl}, \quad (39)$$

$$\bar{R}_{3ijl} = R_{3ijl}, \quad \bar{U}_{ijl} = P^{-T} (P + U_{ijl})^{-1} P^{-1}, \quad \bar{V}_{ijl} = (I + V_{ijl})^{-1}, \quad (40)$$

$$\bar{Q}_1 = P^{-T} Q_1 P^{-1}, \quad \bar{Q}_2 = P^{-T} Q_2, \quad \bar{Q}_3 = Q_3, \quad (41)$$

and then, performing a congruence transformation to (34) by $\text{diag} \{P, I, P, I\}$, we have

$$\begin{bmatrix} \Gamma_{1ijl} & \Gamma_{2ijl} & \varpi A_{ij}^T P & \varpi C_{ij}^T \\ * & \Gamma_{3ijl} & \varpi B_{wi}^T P & \varpi D_{wi}^T \\ * & * & -\bar{U}_{ijl} & 0 \\ * & * & * & -\bar{V}_{ijl} \end{bmatrix} < 0, \quad \forall i_1, i_2, \dots, i_n, k, l, i, j \quad (42)$$

where

$$\begin{aligned}\Gamma_{1ijl} &\triangleq -\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} P + (\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \underline{\epsilon}_{ij i_1 i_2 \dots i_n kl}) R_{1ijl} + \left(\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \frac{1}{pc} \right) Q_1, \\ \Gamma_{2ijl} &\triangleq (\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \underline{\epsilon}_{ij i_1 i_2 \dots i_n kl}) R_{2ijl} + \left(\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \frac{1}{pc} \right) Q_2, \\ \Gamma_{3ijl} &\triangleq -\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} \gamma^2 I + (\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \underline{\epsilon}_{ij i_1 i_2 \dots i_n kl}) R_{3ijl} + \left(\bar{\epsilon}_{ij i_1 i_2 \dots i_n kl} - \frac{1}{pc} \right) Q_3.\end{aligned}$$

Moreover, let the following matrices be partitioned as:

$$Q \triangleq \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix}, \quad R_{ijl} \triangleq \begin{bmatrix} R_{1ijl} & R_{2ijl} \\ * & R_{3ijl} \end{bmatrix}. \quad (43)$$

Then based on inequalities in (42), and by Schur complement, it yields

$$Z_{ij} - \frac{1}{pc} Q < 0, \quad \forall i_1, i_2, \dots, i_n, k, l, i, j,$$

thus,

$$\sum_{i=1}^p \sum_{j=1}^c Z_{ij} - Q < 0, \quad \forall i_1, i_2, \dots, i_n, k, l,$$

which meets the condition (13) in Theorem 1.

Similarly, for the set of inequalities in (35), letting $\bar{X}_{ijl} = P^{-T} (X_{ijl} - P)^{-1} P^{-1}$ and $\bar{Y}_{ijl} = (Y_{ijl} - I)^{-1}$ with matrices in (43), then performing a congruence transformation to (35) by $\text{diag}\{P, I, P, I\}$, one can obtain the condition (14) in Theorem 1. Thus, the condition in (35) is satisfied.

Meanwhile, by performing congruence transformations to (36) and (37) by $\text{diag}\{I, P\}$ and $\text{diag}\{P, P\}$, respectively, (17) and (18) in Theorem 1 can be obtained, respectively. Hence, all the conditions in Theorem 2 satisfy the conditions of Theorem 1. The proof is completed. ■

Remark 3: From Theorem 2, the existence condition of the desired IT2 fuzzy controller in (4) is presented for discrete-time IT2 fuzzy system in (11). The advantage of the presented controller subject the LMIs constraint can guarantee the poles of the real systems rest in a given disk, which is according to the need of engineering in practical applications.

Remark 4: To consider more, the computational complexity can be calculated for the results. For Theorem 2, the number of the LMIs is $2\{1, 2, \dots, n\}pcq(\vartheta + 1) + pc(\vartheta + 1) + p + pc$ with $7pc(\vartheta + 1) + 7$ decision variables, in which the maximum number of the LMIs is $2npcq(\vartheta + 1) + pc(\vartheta + 1) + p + pc$ if n system states are chosen to be divided.

IV. PRACTICAL EXAMPLE

In this section, two simulation examples are provided to validate the effectiveness of the designed control scheme proposed in this paper.

Example 1: In this example, we will use the inverted pendulum example to verify the advantages over the existing type-1 T-S fuzzy model approach [43], and further validate the effectiveness of the optimal control design method. Fig. 1 shows the sketch of the inverted pendulum on a cart.

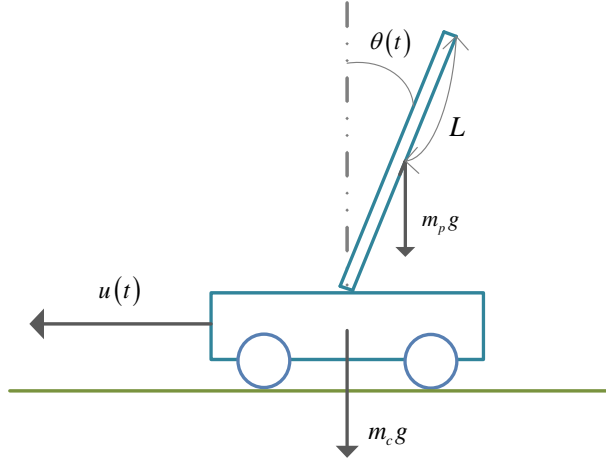


Fig. 1. Diagrammatic sketch of the inverted pendulum on a cart.

The dynamical equation of the inverted pendulum is given below:

$$\ddot{\xi}(t) = \frac{g \sin(\xi(t)) - am_p L \left(\dot{\xi}(t)\right)^2 \sin(2\theta(t))/2 - a \cos(\xi(t)) u(t)}{4L/3 - am_p L \cos^2(\xi(t))} \quad (44)$$

where $\xi(t)$ denotes the angular displacement of the pendulum, $2L = 1$ m is the length of the pendulum, the gravity acceleration is $g = 9.8$ m/s², m_p denote the mass of the pendulum, m_c denote the mass of the cart, $a = 1/(m_p + m_c)$, and $u(t)$ denote the force applied to the cart. We mark $x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T = \begin{bmatrix} \xi(t) & \dot{\xi}(t) \end{bmatrix}^T$. Firstly, we use a type-1 T-S fuzzy system to model the dynamical equations (44) for the comparison with the presented control approach. In such case, $m_p = 2$ kg and $m_c = 4$ kg. A two-rule T-S fuzzy system is established below.

Fuzzy Rule 1 : IF $\xi(t)$ is about $\pm \frac{3}{8}\pi$, THEN

$$\dot{x}(t) = A_1 x(t) + B_1 u(t),$$

Fuzzy Rule 2 : IF $\xi(t)$ is about 0, THEN

$$\dot{x}(t) = A_2 x(t) + B_2 u(t),$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ f_1(\frac{3}{8}\pi) & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ f_1(0) & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & f_2(\frac{3}{8}\pi) \end{bmatrix}^T, B_2 = \begin{bmatrix} 0 & f_2(0) \end{bmatrix}^T,$$

with

$$\begin{aligned} f_1(x_1(t)) &= \frac{g - am_p L x_2^2(t) \cos(x_1(t))}{4L/3 - am_p L \cos^2(x_1(t))} \left(\frac{\sin(x_1(t))}{x_1(t)} \right), \\ f_2(x_1(t)) &= \frac{-a \cos(x_1(t))}{4L/3 - am_p L \cos^2(x_1(t))} \left(\frac{\sin(x_1(t))}{x_1(t)} \right). \end{aligned}$$

In order to obtain the discrete-time form of the fuzzy system above, we let time period $T = 0.01$ s. Then, considering the disturbance input, the fuzzy system of discrete-time form (see [43], system (8–9)) is established below:

$$\begin{cases} x(k+1) = \sum_{i=1}^r h_i(\xi(k)) [A_i x(k) + B_i u(k) + B_{wi} w(k)], \\ z(k) = \sum_{i=1}^r h_i(\xi(k)) [C_i x(k) + D_i u(k) + D_{wi} w(k)], \end{cases} \quad (45)$$

where the corresponding system parameters (annotated in [43], (1–3) and (8–9)), are given as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} 1.0006 & 0.0100 \\ 0.1186 & 1.0006 \end{bmatrix}, A_2 = \begin{bmatrix} 1.0000 & 0.0100 \\ -0.0010 & 1.0000 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 9.0470 \times 10^{-4} & 1.8096 \times 10^{-1} \end{bmatrix}^T, B_2 = \begin{bmatrix} -1.5385 \times 10^{-5} & -3.0769 \times 10^{-3} \end{bmatrix}^T, \end{aligned}$$

with the considered matrices

$$C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, B_{w1} = B_{w2} = \begin{bmatrix} 0 & 0.02 \end{bmatrix}^T, D_1 = D_2 = 1, D_{w1} = D_{w2} = 0.01,$$

and the grade of membership functions are given by

$$\begin{aligned} h_1(\xi(k)) &= \left(1 - \frac{1}{1 + \exp(3(\xi(k) - \pi/2))} \right) \times \frac{1}{1 + \exp(3(\xi(k) + \pi/2))}, \\ h_2(\xi(k)) &= 1 - h_1(\xi(k)). \end{aligned}$$

Next, for further comparing with the presented control approach, we apply the Corollary 3 in [43] to obtain the control gains. Moreover, we constrain an input condition $u(k) \leq u_{\max} = 1.2$ with parameter $\rho = 1$ (the condition can be derived directly as the proof of (36)). Simultaneously, we choose the same

disturbance attenuation index $\gamma = 0.6439$, which is obtained from the Theorem 2 of the this paper in the following context. Then the control gains can be obtained as follows.

$$K_1 = \begin{bmatrix} -0.3244 & -6.4246 \end{bmatrix}, K_2 = \begin{bmatrix} -2.3126 & -0.0443 \end{bmatrix}.$$

Assume the initial state $x(0) = \begin{bmatrix} \frac{\pi}{12} & 0 \end{bmatrix}^T$, and a disturbance input $w(k) = 1/(2\sin(k) + e^k)$. The controller (see [43], (11)) is employed to control the system in (45). Thus, the simulation results are provided in Figs. 2 and 3. The system state responses are depicted in Fig. 2, and the control force is shown in Fig. 3.

On the other hand, we consider the parametric uncertainties m_p and m_c existing in the pendulum system satisfying $m_{p\min} = 1 \text{ kg} \leq m_p \leq m_{p\max} = 3 \text{ kg}$, $m_{c\min} = 3 \text{ kg} \leq m_c \leq m_{c\max} = 5 \text{ kg}$. We use a four-rule IT2 fuzzy model below to describe the inverted pendulum subject to parametric uncertainties.

Fuzzy Rule i : IF $f_1(t)$ is M_1^i and $f_2(t)$ is M_2^i , THEN

$$x(t) = A_i x(t) + B_i u(t), \quad (46)$$

where

$$\begin{aligned} A_1 &= A_2 = \begin{bmatrix} 0 & 1 \\ f_{1\min} & 0 \end{bmatrix}, A_3 = A_4 = \begin{bmatrix} 0 & 1 \\ f_{1\max} & 0 \end{bmatrix}, \\ B_1 &= B_3 = \begin{bmatrix} 0 & f_{2\min} \end{bmatrix}^T, B_2 = B_4 = \begin{bmatrix} 0 & f_{2\max} \end{bmatrix}^T. \end{aligned}$$

Assume that the inverted pendulum operates in the workplace described by $x_1 = \xi(t) \in [-3\pi/8, 3\pi/8]$ and $x_2 = \xi(t) \in [-3, 3]$. Thus, $f_{1\min} = 11.1261$, $f_{1\max} = 21.3333$, $f_{2\min} = -0.4615$, and $f_{2\max} = -0.0748$. In order to obtain the discrete-time form of IT2 fuzzy model and compare with the type-1 in (45), we use the same sampling time period to obtain the discrete-time IT2 fuzzy system with disturbance input in (2), which is expressed below.

Fuzzy Rule i : IF $\xi(t)$ is M_1^i and $\xi(t)$ is M_2^i , THEN

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k) + B_{wi} w(k), \\ z(k) = C_i x(k) + D_i u(k) + D_{wi} w(k), \end{cases} \quad (47)$$

where

$$\begin{aligned} A_1 &= A_2 = \begin{bmatrix} 1.0006 & 0.0100 \\ 0.1113 & 1.0006 \end{bmatrix}, A_3 = A_4 = \begin{bmatrix} 1.0012 & 0.0100 \\ 0.2353 & 1.0012 \end{bmatrix}, \\ B_1 &= B_3 = \begin{bmatrix} -0.23 \times 10^{-4} & -0.46 \times 10^{-2} \end{bmatrix}^T, B_2 = B_4 = \begin{bmatrix} -0.37 \times 10^{-5} & -0.75 \times 10^{-3} \end{bmatrix}^T, \end{aligned}$$

TABLE I
LOWER AND UPPER MEMBERSHIP FUNCTIONS

Lower membership functions of the pant	Upper membership functions of the pant
$\underline{\mu}_{M_1^1}(x_1) = 1 - \exp\left(-\frac{x_1^2}{1.5}\right)$	$\bar{\mu}_{M_1^1}(x_1) = 0.25 \exp\left(-\frac{x_1^2}{0.3}\right)$
$\underline{\mu}_{M_1^2}(x_1) = \underline{\mu}_{M_1^1}(x_1)$	$\bar{\mu}_{M_1^2}(x_1) = \bar{\mu}_{M_1^1}(x_1)$
$\underline{\mu}_{M_1^3}(x_1) = 1 - \bar{\mu}_{M_1^1}(x_1)$	$\bar{\mu}_{M_1^3}(x_1) = 1 - \underline{\mu}_{M_1^1}(x_1)$
$\underline{\mu}_{M_1^4}(x_1) = \underline{\mu}_{M_1^3}(x_1)$	$\bar{\mu}_{M_1^4}(x_1) = \bar{\mu}_{M_1^3}(x_1)$
$\underline{\mu}_{M_2^1}(x_1) = 0.4 \exp\left(-\frac{x_1^2}{0.2}\right)$	$\bar{\mu}_{M_2^1}(x_1) = \exp\left(-\frac{x_1^2}{2.5}\right)$
$\underline{\mu}_{M_2^2}(x_1) = 1 - \bar{\mu}_{M_2^1}(x_1)$	$\bar{\mu}_{M_2^2}(x_1) = 1 - \underline{\mu}_{M_2^1}(x_1)$
$\underline{\mu}_{M_2^3}(x_1) = \underline{\mu}_{M_2^2}(x_1)$	$\bar{\mu}_{M_2^3}(x_1) = \bar{\mu}_{M_2^2}(x_1)$
$\underline{\mu}_{M_2^4}(x_1) = \underline{\mu}_{M_2^3}(x_1)$	$\bar{\mu}_{M_2^4}(x_1) = \bar{\mu}_{M_2^3}(x_1)$
Lower membership functions of the controller	Upper membership functions of the controller
$\underline{\mu}_{N_1^1}(x_1) = \exp\left(-\frac{x_1^2}{0.5}\right)$	$\bar{\mu}_{N_1^1}(x_1) = \underline{\mu}_{N_1^1}(x_1)$
$\underline{\mu}_{N_1^2}(x_1) = \underline{\mu}_{N_1^1}(x_1)$	$\bar{\mu}_{N_1^2}(x_1) = \underline{\mu}_{N_1^2}(x_1)$

with the comparative matrices

$$C_1 = C_2 = C_3 = C_4 = \begin{bmatrix} 0 & 1 \end{bmatrix}^T, D_1 = D_2 = D_3 = D_4 = 1,$$

$$B_{w1} = B_{w2} = B_{w3} = B_{w4} = \begin{bmatrix} 0 & 0.020 \end{bmatrix}^T, D_{w1} = D_{w2} = D_{w3} = D_{w4} = 0.010.$$

Besides, the lower and upper membership functions are given in Table I, and for generality, we set $\underline{\alpha}_i(\xi(k)) \in [0, 1]$ ($\bar{\alpha}_i(\xi(k)) = 1 - \underline{\alpha}_i(\xi(k))$) for $i = 1, 2, 3, 4$, which obey the Gauss distribution and satisfy $\sum_{i=1}^4 \phi_i(\xi(k)) = 1$ to describe the parametric uncertainty. We use a two-rule IT2 fuzzy controller to stabilize the unstable system (47) via the lower and upper membership functions chosen in Table I and we choose $\underline{\beta}_j(x_1) = \bar{\beta}_j(x_1) = 0.5$ for simplicity.

We use one sub-FOU (i.e., $\tau = 0, l = 1$) and divide the state x_1 into 2000 equal-size sub-states (i.e., $k = 1, 2, \dots, 2000$). Thus, the upper and lower bounds of k -th state $x_1^{k,l}$ in the FOU l are defined as $\underline{x}_1^{k,l} = (3\pi/4)/2000(k - 101)$, $\bar{x}_1^{k,l} = (3\pi/4)/2000(k - 100)$. Then the constant scalars are determined by $\underline{\epsilon}_{ij1k1} = \phi_i(\underline{x}_1^{k,l})\psi_j(\underline{x}_1^{k,l})$, $\underline{\epsilon}_{ij2k1} = \phi_i(\bar{x}_1^{k,l})\psi_j(\bar{x}_1^{k,l})$, $\bar{\epsilon}_{ij1k1} = \bar{\phi}_i(\underline{x}_1^{k,l})\bar{\psi}_j(\underline{x}_1^{k,l})$, $\bar{\epsilon}_{ij2k1} = \bar{\phi}_i(\bar{x}_1^{k,l})\bar{\psi}_j(\bar{x}_1^{k,l})$. Moreover, the lower and upper membership functions \underline{h}_{ij1} and \bar{h}_{ij1} in the form of (8) and (9) are defined by choosing $v_{11k1}(x_1) = 1 - (x_1 - \underline{x}_1^{k,l}) / (\bar{x}_1^{k,l} - \underline{x}_1^{k,l})$ and $v_{12k1}(x_1) = 1 - v_{11k1}(x_1)$, respectively. We remove the constraint (37) and use the same input constraint (36) for comparing with the type-1 one.

Hence, the parameters for Theorem 2 are ready to derive the controller gains in the form of (38). By using the Robust Control Toolbox in MatLab, a feasible solution for controller gains via Theorem 2 is listed as follows:

$$G_1 = \begin{bmatrix} 340.6544 & 77.4515 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 340.6561 & 77.4520 \end{bmatrix}, \quad (48)$$

and the \mathcal{H}_∞ performance index is $\gamma = 0.6439$.

Next, based on the gains in (48), we depict the responses of the closed-loop system under the same initial state and the same random disturbance input. Figs. 4 and 5 show the main simulation results. The system state responses are depicted in Fig. 4, and the control force is shown in Fig. 5. Apparently, the presented control method shows the superiority than the type-1 one.

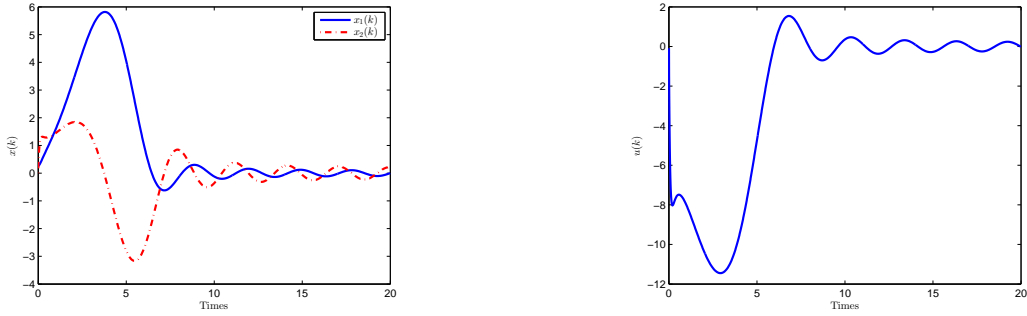


Fig. 2. System state responses under the type-1 case in [43]. Fig. 3. Control input under the type-1 case in [43].

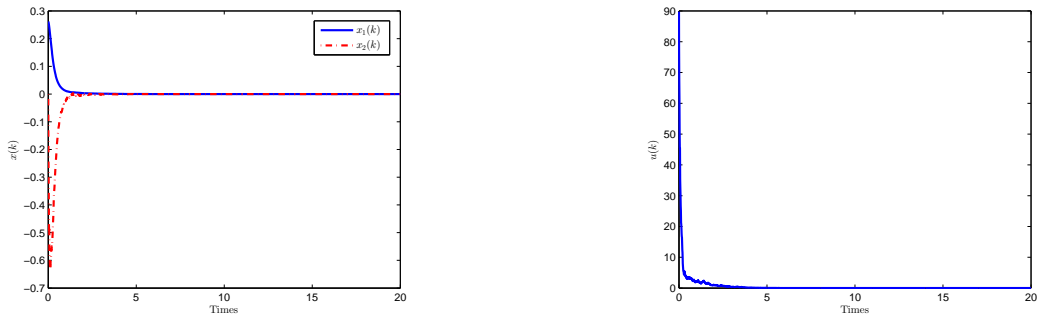


Fig. 4. System state responses under the presented IT2 case. Fig. 5. Control input under the presented IT2 case.

In addition, we will append the constraint (37) to testify the presented optimal control scheme with the \mathcal{D} -stability constraint in the paper, and make a comparison with the removed one. For simplicity, we

divide the state x_1 into 200 equal-size sub-states (i.e., $k = 1, 2, \dots, 200$) in this case. We reset poles being in the disk region $\mathcal{D}(0.3, 0.6)$. Under the same conditions, via Theorem 2, the controller gains can be obtained as

$$G_1 = \begin{bmatrix} 2817.4000 & 281.4683 \end{bmatrix}, G_2 = \begin{bmatrix} 2817.7519 & 281.5026 \end{bmatrix}. \quad (49)$$

Thus, based on the gains in (49), the poles of four LTI systems and two controllers are shown in Fig. 6, in which all poles are assigned in given disk region. Also, under the same conditions given above, the compared simulation results are provided in Figs. 7 and 8, which are depicted the different state trajectories of the two different cases. Obviously, the system responses with constraint case perform preferable than the one without the \mathcal{D} -stability constraint, which has also demonstrated the effectiveness of the presented optimal control scheme.

Example 2: A numerical example is given in this example to further illustrate the effectiveness of the proposed control scheme. The system matrices of 4-rule fuzzy system with a sampling period $T = 0.01$ s in the form of (1) are given as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} 1.0001 & 0.0080 & 0 \\ 0.0200 & 1.6000 & 0.0100 \\ 0.0011 & 0.1100 & 1.0006 \end{bmatrix}, A_2 = \begin{bmatrix} 1.0001 & 0.0080 & 0 \\ 0.0200 & 1.0006 & 0.0100 \\ 0.0011 & 0.1100 & 1.0006 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 1.0000 & 0.0090 & 0 \\ 0 & 1.0012 & 0.0100 \\ 0 & 0.2351 & 1.0012 \end{bmatrix}, A_4 = \begin{bmatrix} 1.0000 & 0.0090 & 0 \\ 0 & 1.0012 & 0.0100 \\ 0 & 0.2351 & 1.0012 \end{bmatrix}, \\ B_1 &= B_3 = \begin{bmatrix} 0 & 0 & -0.0046 \end{bmatrix}^T, B_2 = B_4 = \begin{bmatrix} 0 & 0.014 & 0.0008 \end{bmatrix}^T, \\ B_{w1} &= B_{w2} = B_{w3} = B_{w4} = \begin{bmatrix} 0 & 0.020 & 0 \end{bmatrix}^T, \\ C_1 &= C_2 = C_3 = C_4 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T, \\ D_1 &= D_2 = D_3 = D_4 = 1, D_{w1} = D_{w2} = D_{w3} = D_{w4} = 0.010. \end{aligned}$$

The membership functions are chosen as the same ones in the Table I with the $x_1 \in [-4, 4]$. Fig. 9 shows that the open-loop system is unstable. We give the constraints $u(k) \leq u_{\max} = 700$ with parameter $\rho = 1$ and the \mathcal{D} -stability constraint with poles being in the disk region $\mathcal{D}(0.75, 0.25)$. We use one sub-FOU (i.e., $\tau = 0, l = 1$) and divide the state x_1 into 1000 equal-size sub-states (i.e., $k = 1, 2, \dots, 1000$). Thus, the upper and lower bounds of k -th state $x_1^{k,l}$ in the FOU l are defined as $\underline{x}_1^{k,l} = 8/2000(k - 1001)$, $\bar{x}_1^{k,l} = 8/2000(k - 1000)$. Then the constant scalars are determined by

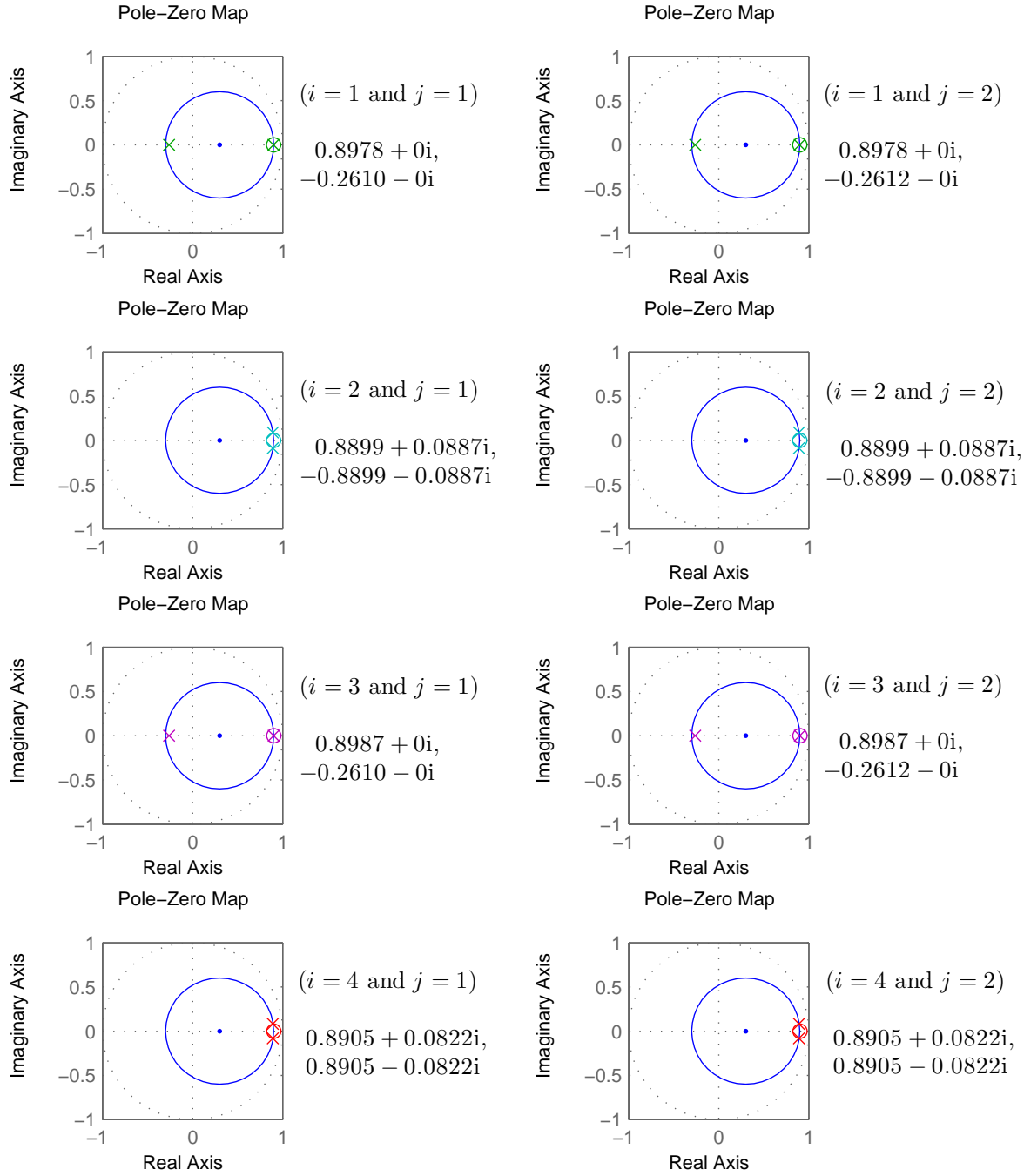


Fig. 6. Poles of the closed-loop system ("o" denotes a zero point, and "x" denotes a pole)

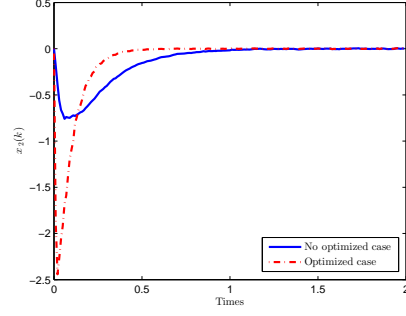
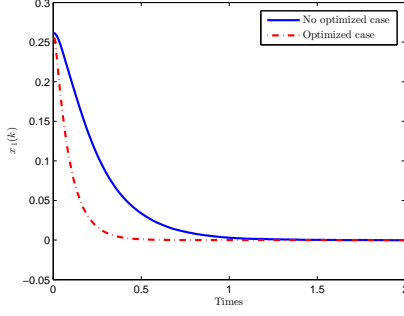


Fig. 7. Simulation compared result of state response $x_1(k)$. Fig. 8. Simulation compared result of state response $x_2(k)$.

$\underline{\epsilon}_{ij1k1} = \underline{\phi}_i(\underline{x}_1^{k,l})\underline{\psi}_j(\underline{x}_1^{k,l})$, $\underline{\epsilon}_{ij2k1} = \underline{\phi}_i(\overline{x}_1^{k,l})\underline{\psi}_j(\overline{x}_1^{k,l})$, $\overline{\epsilon}_{ij1k1} = \overline{\phi}_i(\underline{x}_1^{k,l})\overline{\psi}_j(\underline{x}_1^{k,l})$, $\overline{\epsilon}_{ij2k1} = \overline{\phi}_i(\overline{x}_1^{k,l})\overline{\psi}_j(\overline{x}_1^{k,l})$. Moreover, the lower and upper membership functions \underline{h}_{ij1} and \overline{h}_{ij1} in the form of (8) and (9) are defined by choosing $v_{11k1}(x_1) = 1 - (x_1 - \underline{x}_1^{k,l}) / (\overline{x}_1^{k,l} - \underline{x}_1^{k,l})$ and $v_{12k1}(x_1) = 1 - v_{11k1}(x_1)$, respectively. Then by Theorem 2, we obtained the controller gains in the form of (38) as follows:

$$G_1 = \begin{bmatrix} 185.7230 & 401.7096 & 84.4656 \end{bmatrix}, G_2 = \begin{bmatrix} 185.7657 & 401.7635 & 84.4791 \end{bmatrix}, \quad (50)$$

and the \mathcal{H}_∞ performance index is $\gamma = 23.8038$. Assume the disturbance input $w(k) = 1 / (2 \sin(k) + e^k)$. Thus, based on the gains in (50), Fig. 12 shows the poles of the closed-loop systems composed of four LTI systems and two controllers, in which all poles are assigned in given disk region. The closed-loop system states trajectories are depicted in Fig. 10 with the control input given in Fig. 11. The simulation results illustrate the effectiveness of the proposed control scheme in this paper.

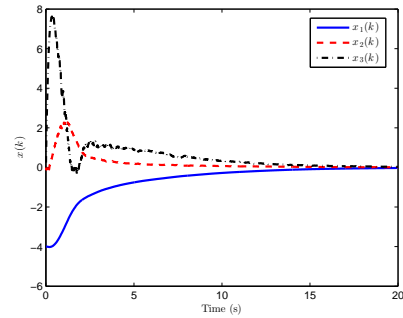
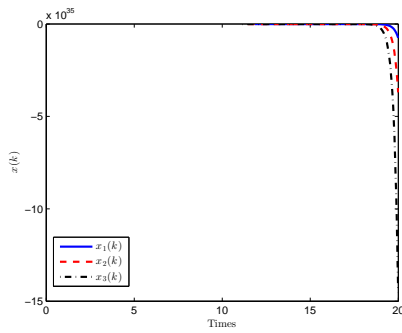


Fig. 9. State responses of open-loop system in Example 2. Fig. 10. State responses of closed-loop system in Example 2.

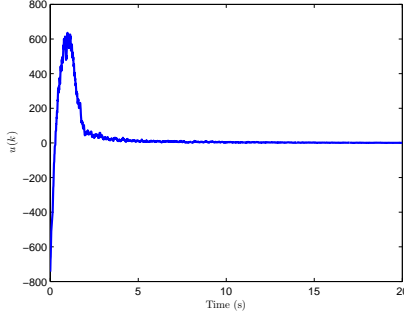
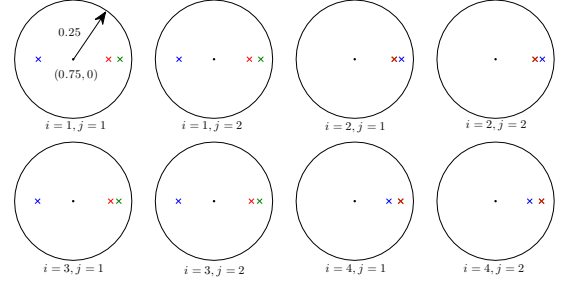


Fig. 11. Control input of closed-loop system in Example 2.

Fig. 12. Poles in disk $\mathcal{D}(0.75, 0.25)$ for the eight sub-closed-loop systems in Example 2.

V. CONCLUSION

This paper has solved the problem of IT2 state-feedback control design for discrete-time IT2 fuzzy systems. The IT2 fuzzy systems have been considered with input constraint and \mathcal{D} -stability constraint. Sufficient criterions of asymptotic stability with \mathcal{H}_∞ performance have been given for the discrete-time IT2 fuzzy systems with input and \mathcal{D} -stability constraints. The fuzzy state-feedback controller has been designed such that the resulted closed-loop system with input constraint is asymptotically stable with \mathcal{H}_∞ performance, and the poles of the closed-loop system are all rested in a desired disk region. The inverted pendulum system has demonstrated the effectiveness and superiority of the presented optimal control design method.

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